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UNSTEADY FLUID FLOW IN TUBES AND THIN-WALL ENVELOPES

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Classical and new mathematical models describing the phenomenon of hydraulic shock are considered.

In paper [1], using the equations of a hydraulic shock, unsteady fluid flow in tubes and in a system of pipelines is studied. A bleeding pipe has an expansion bend with compressed air to reduce the pressure increase in a hydraulic shock. These hydrosystems are found, for example, in hydraulic presses and pressure pipelines of hydroelectric power plants with a pneumatic compensation reservoir. An analytical graphical solution of the problem was executed by the well-known Schneider-Bergeron method. In 1961 the author, together with A. A. Atavin, performed a series of computer calculations for the given hydraulic system using the method of characteristics. The method of characteristics is convenient for solving differential equations of unsteady fluid flow in pressure pipelines since it makes it possible to neglect convective terms in the equations, thus considerably simplifying the calculation. In [1], due to the high complexity of the method, calculations were performed only for the case of a single turning-off of the shut-off device. In actual practice turning-off and turning-on of the shut-off device has a periodic character. Moreover, the Schneider-Bergeron method itself requires the introduction of some assumptions into the computational scheme, in particular, all hydraulic resistances are referred to one of the ends of the pipeline. The results of computer calculations showed good agreement with the results of [1] for the case of single turning-off of the shut-off device. With periodic turning-off and turning-on of the shut-off device a resonance effect is observed in the hydraulic system if the duration of the working cycle is close to the period of the system's natural oscillations.

Starting with the classical work of N. E. Zhukovskii (1899), who suggested that convective terms be omitted in the equations of a hydraulic shock, mainly these equations have been studied as yet.

One of the first studies of the nonlinear theory of a hydraulic shock was performed in [2], where a grounded classification of the system of equations for open and pressure flows is investigated. The system of equations of a hydraulic shock suggested in [2] has the form

$$\frac{\partial \rho}{\partial t}+\frac{\partial \rho u}{\partial x}=0\,,\quad \frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+\frac{a^2}{\rho}\frac{\partial \rho}{\partial x}=gi(x)-F(\rho\,,\ x)\,u\,|u|\,. \tag{1}$$

With arbitrary functions $i(x)$ and $F(\rho, x)$, system of Eqs. (1) admits one operator $X_1 = \partial/\partial t$, and group extension takes place at $i(x) = \alpha/x$, $F(\rho, x) = k(\rho)/x$, when $X_2 = t\partial/\partial t + x\partial/\partial x$, and also at $i(x) = \alpha$, $F(\rho, x) = k(\rho)$, when $X_2 = \partial/\partial x$; the widest (infinite) group takes place at $i(x) = 0$, $F(\rho, x) = 0$. Omitting the infinite part of the group, which is not usually used for practical calculations, we have

$$X_1=\frac{\partial}{\partial t}\,,\ \ X_2=\frac{\partial}{\partial x}\,,\ \ X_3=t\frac{\partial}{\partial t}+x\frac{\partial}{\partial x}\,,\ \ X_4=t\frac{\partial}{\partial x}+\frac{\partial}{\partial u}\,,\ \ X_5=\rho\frac{\partial}{\partial \rho}\,. \tag{2}$$

Note that in this case we obtain linear equations by interchanging the positions of the dependent and independent variables

$$\frac{\partial x}{\partial u}-u\frac{\partial t}{\partial u}+\rho\frac{\partial t}{\partial \rho}=0\,,\quad \frac{\partial x}{\partial \rho}-u\frac{\partial t}{\partial \rho}+\frac{a^2}{\rho}\frac{\partial t}{\partial u}=0\,.$$

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But in practice this hodograph method is not used. We shall show how to employ knowledge of group (2).

Self-similar solutions correspond to the extension operator $X_3 + \alpha X_5$ and have the form

$$u = U(\xi), \quad \rho = t^{\alpha} R(\xi), \quad \xi = \frac{x}{t}. \tag{3}$$

Substituting Eqs. (3) into Eq. (1) with a zero right-hand side, we obtain the system of ordinary differential equations

$$(U - \xi) R' + RU' = -\alpha R, \qquad \frac{a^2}{R} R' + (U - \xi) U' = 0. \tag{4}$$

Hence

$$\frac{dU}{d\xi} = \frac{a^2 \alpha}{(U - \xi)^2 - a^2}, \tag{5}$$

i.e., the system of Eqs. (4) is split; first Eq. (5) is solved and then the equation

$$\frac{dR}{d\xi} = -\frac{\alpha R (U - \xi)}{(U - \xi)^2 - a^2}.$$

The solution at $\alpha = 0$ corresponds to the problem of a piston which slides in ($w > 0$) and slides out ($w < 0$) with a constant velocity from a quiescent fluid with density ρ_0 occupying half-space $x > 0$. The solution of this problem has the form

$$\text{when} \quad w < 0 \quad u = \begin{cases} w, \\ \xi - a, \\ 0, \end{cases} \quad \rho = \begin{cases} \rho_0 \exp(w/a), & w < \xi \le w + a, \\ \rho_0 \exp(\xi/a - 1), & w + a < \xi \le a, \\ \rho_0, & \xi > a; \end{cases}$$

$$\text{when} \quad w > 0 \quad u = \begin{cases} w, \\ \\ 0, \end{cases} \quad \rho = \begin{cases} \rho_*, \quad w < \xi < D, & \rho_* = \left(\dfrac{w^2}{2a^2} + \sqrt{\left(\dfrac{w^4}{4a^4} + 1 \right)} \right) \rho_0, \\ \rho_0, \quad \xi > D, & D = (\rho_* w)/(\rho_* - \rho_0). \end{cases}$$

In the second solution the conditions at the discontinuity (a hydroshock wave)

$$D[\rho] = [\rho u], \quad D[\rho u] = [\rho u^2 + a^2 \rho]. \tag{6}$$

Here $[f] = f^+ - f^-$, where f^+ and f^- are the values of the function f on opposite sides of the discontinuity. The conditions (6) are obtained in a regular way from the integral equations of motion (the laws of mass and momentum conservation)

$$\int_C \rho dx - \rho u dt = 0, \quad \int_C \rho u dx - (\rho u^2 + a^2 \rho) dt = \iint_G (gi - Fu|u|) dt dx,$$

where G is a region in plane x, t with the boundary C.

The presented solutions of the problem of a piston glued together give the solution of the problem of the decomposition of an arbitrary discontinuity.

Other invariant solutions are found in a similar manner. For example, the solution of the form

$$u = U(x), \quad \rho = \exp(t + R(x)). \tag{7}$$

corresponds to the operator $X_1 + X_5 = \partial/\partial t + \rho\partial/\partial\rho$. Substituting (7) into system (1) with a zero right-hand side, we obtain the equations

$$1 + UR' + U' = 0, \quad UU' + a^2R' = 0,$$

whose solution yields

$$R = -\frac{1}{2a^2}U^2 + B, \quad \frac{1}{3}U^3 - a^2U = a^2x + A,$$

where A and B are integration constants. If we consider the solutions at $A = 0$ and small x and U, then we obtain a solution of the problem of a fluid flowing into a dead end. In this case pressure grows with time.

The equations of the characteristics of system (1) have the form

$$\frac{dx}{dt} = u \pm a. \tag{8}$$

Since $|u| \ll a$, in the above approximate models $\partial x/\partial t = \pm a$. But when nonlinear terms are ignored the problems of the formation and disappearance of the hydroshock wave (the formation of a discontinuity in the flow region at smooth initial and boundary conditions) are lost.

The problems for one fluid admit generalization to the case of two contacting fluids for each of which the equation of state holds, i.e., $P_i = a_i^2\rho_i$ $(i = 1, 2)$. The conditions on the contact boundary have the form $P^+ = P^-$, $u^+ = u^-$. If a liquid contacts a gas, then on one side of the contact boundary the equations of gasdynamics are solved. This makes it possible to model flow discontinuity (here we shall not dwell on this).

We generalize (1) to the case of cylindrical and spherical symmetry:

$$\frac{\partial\rho}{\partial t} + u\frac{\partial\rho}{\partial r} + \rho\left(\frac{\partial u}{\partial r} + \frac{\nu u}{r}\right) = 0, \quad \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{a^2}{\rho}\frac{\partial\rho}{\partial r} = 0, \tag{9}$$

where $r \geq 0$ is the distance from the axis ($\nu = 1$) or the center ($\nu = 2$) of symmetry.

Equations (9) admit the group

$$X_1 = \frac{\partial}{\partial t}, \quad X_2 = t\frac{\partial}{\partial t} + r\frac{\partial}{\partial r}, \quad X_3 = \rho\frac{\partial}{\partial\rho}.$$

An analog of solution (3) (r is taken instead of x) also leads to the splitting of the solution. First we find $U(\xi)$ from the equation

$$\frac{dU}{d\xi} = \frac{a^2\left(\alpha + \frac{\nu U}{\xi}\right)}{(U-\xi)^2 - a^2}, \tag{10}$$

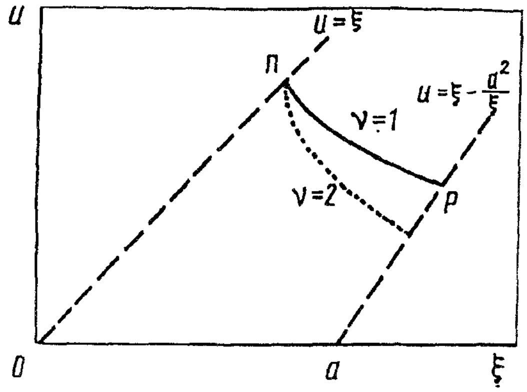
and then $R(\xi)$ from the equation

$$\frac{dR}{d\xi} = -\frac{R\left(\alpha + \frac{\nu U}{\xi}\right)(U-\xi)}{(U-\xi)^2 - a^2}.$$

At $\alpha = 0$ one can formulate the problem of a piston. The space is occupied by a quiescent fluid with density π_0. The amplitude of piston motion with a constant velocity $w = \xi_0 > 0$ begins to increase from the axis ($\nu = 1$) or the point ($\nu = 2$). We have the condition on the piston

$$U = \xi_0, \quad \xi = \xi_0. \tag{11}$$

Equation (10) at $\alpha = 0$ with condition (11) was solved numerically. The solution ended with discontinuity. From conditions (6) at $D = \xi_p$ we obtain the condition



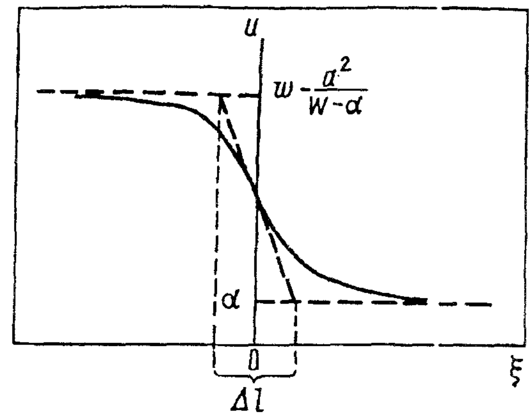


Fig. 1. Numerical solution of problem (10), (11).

Fig. 2. Analytical solution of Eq. (14)

$$U = \xi_p - \frac{a^2}{\xi_p}.$$

A qualitative flow pattern is given in Fig. 1.

Using the experience obtained from studies of hydraulics of open tube beds [3], we generalize the described model of N. E. Zhukovskii to the spatial case, on the one hand, and to the case of allowance for viscosity and dispersion, on the other. Spatial equations of a hydraulic shock were first suggested in [4] and have the form

$$\rho_t + \operatorname{div}(\rho \mathbf{V}) = 0, \quad \mathbf{V}_t + (\mathbf{V}\cdot\nabla)\mathbf{V} + \frac{a^2}{\rho}\nabla\rho = \mathbf{f}. \tag{12}$$

In [4] a group is also presented that is admitted by spatial Eqs. (12) at $\mathbf{f} = 0$. If we take into account fluid viscosity, then the second equation in (12) has the form

$$\mathbf{V}_t + (\mathbf{V}\cdot\nabla)\mathbf{V} + \frac{a^2}{\rho}\nabla\rho - \frac{\mu}{\rho}\Delta\mathbf{V} = \mathbf{f}. \tag{13}$$

We consider the structure of discontinuity (a hydroshock wave), i.e., the solution of one-dimensional Eqs. (13) with $\mathbf{f} = 0$, which depends on $\xi = x - wt$, $w = \text{const}$. Then we obtain ordinary differential equations. In dimensionless variables we assign the conditions: $u = \alpha$ and $\rho = 1$ at $x \to \infty$. Then the continuity equation is integrated and yields

$$\rho = \frac{\alpha - w}{u - w}.$$

Integrating the equation of motion, we obtain

$$(\alpha - w)u + a^2\rho - \mu u' = (\alpha - w)\alpha + a^2.$$

Having excluded ρ and performed easy computations, we have

$$\mu\frac{du}{d\xi} = \frac{\alpha - w}{u - w}(u - \alpha)\left(u - w + \frac{a^2}{w - \alpha}\right), \quad \alpha < u < w - \frac{a^2}{w - \alpha}. \tag{14}$$

The distribution of $u(\xi)$ is given in Fig. 2 in a general form. This solution exists at $w > \alpha + a$. The front width is equal to

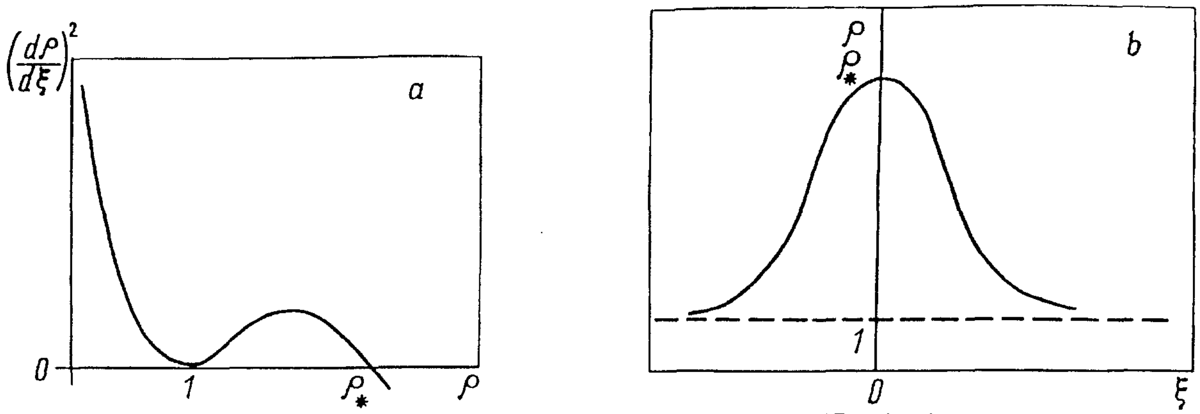


Fig. 3. Graphic representation (a) and solution of Eq. (17) of solution type (b).

$$\Delta l = \frac{w - \alpha - \dfrac{a^2}{w - \alpha}}{\left|\dfrac{du}{d\xi}\right|_{\max}} = \frac{\mu\,[(w-\alpha)^2 - a^2]\,(\alpha\,(w-\alpha) - a^2)}{\alpha\,(w-\alpha)^2\,[w\,(w-\alpha) - a^2]}\,.$$

When fluid and envelope dispersion is taken into account, we obtain the equation of motion

$$\mathrm{V}_t + (\mathrm{V}\cdot\nabla)\,\mathrm{V} + \frac{a^2}{\rho}\nabla\rho + E\nabla A = \mathrm{f}\,, \tag{15}$$

where A can be of the form

$$\text{a)}\ A = \frac{\partial^2\rho}{\partial t^2}\,,\quad \text{b)}\ A = \Delta\rho\quad\text{or}\quad \text{c)}\ A = -\frac{\partial}{\partial t}\,\mathrm{div}\,\mathrm{V}\,. \tag{16}$$

We consider, similarly, a solution of the traveling-wave type for a soliton (a solitary wave) of one-dimensional Eqs. (15) and (16b). The continuity equation coincides with that given above and the equation of motion is written in the form

$$(u - w)\,u' + \frac{a^2}{\rho}\rho' + E\rho''' = 0\,,$$

or, integrating and using the same boundary conditions at infinity,

$$\frac{1}{2}(u - w)^2 + a^2\ln\rho + E\rho'' = \frac{1}{2}(\alpha - w)^2\,.$$

Excluding $u - w$ and integrating once again, we come to the equation

$$\frac{E}{2}\left(\frac{d\rho}{d\xi}\right)^2 = \frac{1}{2}(w-\alpha)^2\rho - a^2\rho\,(\ln\rho - 1) + \frac{1}{2}\frac{(w-\alpha)^2}{\rho} - (w-\alpha)^2 - a^2\,. \tag{17}$$

Its graphic representation for $w > \alpha + a$ and the solution are shown in Fig. 3. One can find a periodic solution in this approximation.

The reduced method of perturbations developed by the author was applied to all of the above-described equations (a generalized representation for open flows is given in [3]).

We consider it using the example of three-dimensional Eqs. (12), (15), and (16a) at f = 0. It is assumed that $\rho = \rho_0$, $u = 0$, $v = 0$, and $w = 0$ for $x \to \infty$. In the first step we substitute independent variables

$$\xi = \varepsilon^b\,(x - \lambda t)\,,\quad \eta = \varepsilon^{b+1/2}y\,,\quad \zeta = \varepsilon^{b+1/2}z\,,\quad \tau = \varepsilon^{b+1}t\,,$$

where ε is a fictitious small parameter necessary for expansions (in final formulas it is assumed that $\varepsilon = 1$) and b, λ are the constants determined below in the process of reduction. Then these equations acquire the form

$$\varepsilon^{b+1}\frac{\partial\rho}{\partial\tau} + \varepsilon^{b}(-\lambda + u)\frac{\partial\rho}{\partial\xi} + \varepsilon^{b+1/2}v\frac{\partial\rho}{\partial\eta} + \varepsilon^{b+1/2}w\frac{\partial\rho}{\partial\zeta} + \rho\left(\varepsilon^{b}\frac{\partial u}{\partial\xi} + \varepsilon^{b+1/2}\frac{\partial v}{\partial\eta} + \varepsilon^{b+1/2}\frac{\partial w}{\partial\zeta}\right) = 0,$$

$$\rho\left[\varepsilon^{b+1}\frac{\partial u}{\partial\tau} + \varepsilon^{b}(-\lambda + u)\frac{\partial u}{\partial\xi} + \varepsilon^{b+1/2}v\frac{\partial u}{\partial\eta} + \varepsilon^{b+1/2}w\frac{\partial u}{\partial\zeta} + \varepsilon^{3b}E\lambda^2\frac{\partial^3\rho}{\partial\xi^3} - \right.$$

$$\left. - \varepsilon^{3b+1}E\lambda\frac{\partial^3\rho}{\partial\xi^2\partial\tau} + \varepsilon^{3b+2}E\frac{\partial^3\rho}{\partial\xi\partial\tau^2}\right] + \varepsilon^{b}a^2\frac{\partial\rho}{\partial\xi} = 0, \qquad (18)$$

$$\rho\left[\varepsilon^{b+1}\frac{\partial v}{\partial\tau} + \varepsilon^{b}(-\lambda + u)\frac{\partial v}{\partial\xi} + \varepsilon^{b+1/2}v\frac{\partial v}{\partial\eta} + \varepsilon^{b+1/2}w\frac{\partial v}{\partial\zeta} + \varepsilon^{3b+1/2}E\lambda^2\frac{\partial^3\rho}{\partial\xi^2\partial\eta} - \right.$$

$$\left. - \varepsilon^{3b+3/2}E\lambda\frac{\partial^3\rho}{\partial\xi\partial\eta\partial\tau} + \varepsilon^{3b+5/2}E\frac{\partial^3\rho}{\partial\eta\partial\tau^2}\right] + \varepsilon^{b+1/2}a^2\frac{\partial\rho}{\partial\eta} = 0,$$

$$\rho\left[\varepsilon^{b+1}\frac{\partial w}{\partial\tau} + \varepsilon^{b}(-\lambda + u)\frac{\partial w}{\partial\xi} + \varepsilon^{b+1/2}v\frac{\partial w}{\partial\eta} + \varepsilon^{b+1/2}w\frac{\partial w}{\partial\zeta} + \varepsilon^{3b+1/2}E\lambda^2\frac{\partial^3\rho}{\partial\xi^2\partial\zeta} - \right.$$

$$\left. - \varepsilon^{3b+3/2}E\lambda\frac{\partial^3\rho}{\partial\xi\partial\zeta\partial\tau} + \varepsilon^{3b+5/2}E\frac{\partial^3\rho}{\partial\zeta\partial\tau^2}\right] + \varepsilon^{b+1/2}a^2\frac{\partial\rho}{\partial\zeta} = 0.$$

In the second step we find a solution in the form of a series with in ε:

$$\rho = \rho_0 + \varepsilon\rho_1(\xi, \eta, \zeta, \tau) + \varepsilon^2\rho_2(\xi, \eta, \zeta, \tau) + \ldots, \quad u = \varepsilon u_1 + \ldots,$$

$$v = \varepsilon^{1/2}[\varepsilon v_1 + \ldots], \quad w = \varepsilon^{1/2}[\varepsilon w_1 + \ldots].$$

After the substitution of these expressions into (10), we require that dispersion terms enter into the second approximation. Then we have $b + 2 = 3b + 1$. Hence, $b = 1/2$. Collecting the terms at small powers of ε, we obtain the equations

$$-\lambda\frac{\partial\rho_1}{\partial\xi} + \rho_0\frac{\partial u_1}{\partial\xi} = 0, \quad -\rho_0\lambda\frac{\partial u_1}{\partial\xi} + a^2\frac{\partial\rho_1}{\partial\xi} = 0,$$

$$-\rho_0\lambda\frac{\partial v_1}{\partial\xi} + a^2\frac{\partial\rho_1}{\partial\eta} = 0, \quad -\rho_0\lambda\frac{\partial w_1}{\partial\xi} + a^2\frac{\partial\rho_1}{\partial\zeta} = 0.$$

From the first two equations we have $\lambda^2 = a^2$. For the leading front we choose $\lambda = a$. Integrating the last equations and taking into account the conditions at infinity, we obtain

$$u_1 = \frac{a}{\rho_0}\rho_1, \quad v_1 = \frac{a}{\rho_0}\int\frac{\partial\rho_1}{\partial\eta}d\xi, \quad w_1 = \frac{a}{\rho_0}\int\frac{\partial\rho_1}{\partial\zeta}d\xi. \qquad (19)$$

In a second approximation from the first two equations of (18) we obtain

$$\frac{\partial\rho_1}{\partial\tau} - a\frac{\partial\rho_2}{\partial\xi} + u_1\frac{\partial\rho_1}{\partial\xi} + \rho_1\frac{\partial u_1}{\partial\xi} + \rho_0\left(\frac{\partial u_2}{\partial\xi} + \frac{\partial v_1}{\partial\eta} + \frac{\partial w_1}{\partial\zeta}\right) = 0,$$

$$\rho_0\left(\frac{\partial u_1}{\partial\tau} - a\frac{\partial u_2}{\partial\xi} + u_1\frac{\partial u_1}{\partial\xi} + a^2E\frac{\partial^3\rho_1}{\partial\xi^3}\right) - \rho_1 a\frac{\partial u_1}{\partial\xi} + a^2\frac{\partial\rho_2}{\partial\xi} = 0.$$

After the substitution of expressions (19) into these equations, we write

$$2\frac{\partial\rho_1}{\partial\tau} + 2\frac{a}{\rho_0}\rho_1\frac{\partial\rho_1}{\partial\xi} + \rho_0 aE\frac{\partial^3\rho_1}{\partial\xi^3} + a\int\left(\frac{\partial^2\rho_1}{\partial\eta^2} + \frac{\partial^2\rho_1}{\partial\zeta^2}\right)d\xi = 0,$$

and after the differention with respect to ξ

$$\frac{\partial}{\partial\xi}\left(2\frac{\partial\rho_1}{\partial\tau} + 2\frac{a}{\rho_0}\rho_1\frac{\partial\rho_1}{\partial\xi} + \rho_0 aE\frac{\partial^3\rho_1}{\partial\xi^3}\right) + a\left(\frac{\partial^2\rho_1}{\partial\eta^2} + \frac{\partial^2\rho_1}{\partial\zeta^2}\right) = 0.$$

This is the well-known Kadomtsev-Petviashvili equation. At $E = 0$ we obtain the Khokhlov-Zabolotskii equation, to which Eqs. (12) are reduced.

In a similar manner we obtain for Eqs. (13) $b = 1$ and

$$\frac{\partial}{\partial\xi}\left(2\frac{\partial\rho_1}{\partial\tau} + 2\frac{a}{\rho_0}\rho_1\frac{\partial\rho_1}{\partial\xi} + \rho_0 a\mu\frac{\partial^2\rho_1}{\partial\xi^2}\right) + a\left(\frac{\partial^2\rho_1}{\partial\eta^2} + \frac{\partial^2\rho_1}{\partial\zeta^2}\right) = 0.$$

This is called the Khokhlov-Zabolotskii-Kuznetsov equation. In theone-dimensional case the expression within the brackets equals zero. This is the Burgers and Corteveg-de-Vries equation of simple waves.

All the presented nonlinear evolution equations describe the distribution of a package of waves of a small but finite amplitude. Hence it can be concluded that these waves propagate with a constant velocity, but their amplitude changes nonlinearly in accordance with the solution of this equation.

On the basis of Eqs. (1), (9), (12), (13), (15), and (16), numerical calculations of one- and two-dimensional problems (transition of a hydroshock wave from a wide channel to a narrow one and, conversely, interaction of a hydroshock wave with bodies of a simple shape (wedge, round cylinder, etc.)) were performed on a computer. Here both the methods of through calculation and the methods of separation of discontinuties were used.

System of Eqs. (1) at $i = \text{const}$ and $F(\rho, x) = K(\rho)$ $(gi - K(\rho_0)\alpha^2 = 0)$ is reduced to the equation

$$2\frac{\partial\rho_1}{\partial\tau} + 2\frac{a}{\rho_0}\rho_1\frac{\partial\rho_1}{\partial\xi} = \begin{cases} -\dfrac{\rho_0\alpha}{a}\left[2\dfrac{K(\rho_0)\,a}{\rho_0} + \alpha K'(\rho_0)\right]\rho_1, & i \neq 0 \quad \left(b = -1,\ \lambda = \alpha + a,\ u_1 = \dfrac{a}{\rho_0}\rho_1\right), \\ -\dfrac{aK(\rho_0)}{\rho_0}\rho_1\,|\rho_1|, & i = 0 \quad (\alpha = 0,\ b = 0,\ \lambda = a). \end{cases}$$

System of Eqs. (9) is reduced to the equation

$$2\frac{\partial\rho_1}{\partial\tau} + 2\frac{a}{\rho_0}\rho_1\frac{\partial\rho_1}{\partial\xi} + \frac{\nu\rho_1}{\tau} = 0 \quad (b - \text{any},\ \alpha = 0,\ \lambda = 0).$$

This shows the validity of the reduced method of perturbations.

NOTATION

ρ, fluid density; $\mathbf{V} = (u, v, w)$, fluid velocity vector; a, velocity of small perturbations in the system fluid–thin-wall envelope; D, discontinuity velocity; w, piston velocity; ν, coefficient of flow symmetry; g, gravity acceleration; i, inclination of tube axis; F, coefficient of hydraulic (turbulent) friction; $\mathbf{f}$, vector of mass forces; μ, coefficient of fluid viscosity; E, coefficient of dispersion; ε, fictitious small parameter; ξ, η, ζ, λ, radial variables; x, y, z, Cartesian coordinates; t, time.

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